Optimizing Navy Mission Planning

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ABSTRACT

Navy Mission Planner suggests logistically supportable ship employment plans to maximize anticipated military mission accomplishment. An oceanic area of operations is parceled into homogeneous regions, and day by day there are missions located in many of these regions (e.g., antisubmarine warfare, air defense, and maritime interdiction). Some missions may necessitate that other missions be assigned simultaneously in the same region (e.g., mine clearance may require air defense protection), and some prerequisite missions may need to be completed in advance (e.g., antisubmarine warfare a day before a port breakout). Each ship, whether a combatant, unarmed naval ship, or supply vessel, can be operated in any one of what we call its alternate combined mission capability sets, wherein it can complete a set of multiple missions simultaneously, albeit with varying effectiveness depending on the mission set undertaken and readiness condition of the ship. Our planner can integrate logistic ship operations to support combat missions. However, these and other unarmed ships may need combatant escorts that can be shared region-wide, or must keep close. A typical scenario involves about 20 regions, a 15-day planning horizon, 300 missions, and 30 ships. The goal is a responsive, intuitive operational planning assistant.

> *Or to take arms against a sea of troubles,*
> *and by opposing end them.*
>
> —Hamlet III-1 Shakespeare

INTRODUCTION

Navy Mission Planner is an optimization-based decision support system to help operational planners characterize an anticipated conflict scenario and create reasonably detailed logistically supported employment plans for coordinated participation of ships over time.

The model presented here derives from the purely combatant planner introduced by Dugan (2007), the embellished version contributed by Silva (2009), a generalization to include logistics (followed closest here) by Hallmann (2009), and recent work by Baker (2019). The logistic portion of the new planning tool is inspired by the Combat Logistics Force (CLF) planner by Brown and Carlyle (2008)

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APPLICATION AREAS:
Air and Missile Defense; Littoral Warfare and Regional Sea Control; Strike Warfare; Computing Advances in Military

OR METHODS:
Linear Integer Programming

and the more recent Replenishment at Sea Planner by Brown et al. (2017) (see this for references to open literature and comparison with planning of civil ocean shipping). There are only a few open literature publications about (optimally) scheduling our naval combatants, yet these are hugely expensive national assets and best use of them is of strategic importance to us. Brown et al. (1990) schedule Atlantic Fleet surface combatants with weekly resolution for a year, and Brown et al. (1996) schedule Coast Guard Cutters with weekly time resolution over a planning quarter. There are a number of simulation models of naval operations, but we restrict our interest to optimization-based decision support. We want to specify the limitations on our courses of action, and discover from our model the best way to plan operations.

The relevance of such a model derives from the frequency with which such operational plans must be prepared and amended, responding to developing world events.

Navy Mission Planner anticipates three levels of advice:

1. The least complicated anticipates a set of spatially diverse missions in an area of responsibility (AOR), each with an anticipated execution date over the next few weeks. These missions are to be completed by Navy combatants (e.g., cruisers [CGs], destroyers [DDGs], littoral combat ships [LCSs], etc.) as these ships arrive day by day in the AOR. Geography is important, and the transit time from one location to another to complete various missions is a key constraint. Missions also have dependencies among them. For instance, an Integrated Air and Missile Defense (IAMD) mission may be required in some particular location to cover an antisubmarine warfare (ASW) mission. Combatants are capable of performing more than one mission simultaneously, but with varying degrees of effectiveness depending on the simultaneous mission mix and the particular combatant's readiness, training levels, and weapon inventory. Combatants must be scheduled for necessary logistics (e.g., refueling) missions at particular locations and times. These may occur in port (INREP), or underway at logistics "gas stations" (UNREP) located at sea, but not necessarily collocated with combat missions.
2. The next level of complexity considers supplying combatants from CLF supply ships (e.g., T-AKE, T-AO, T-AOE) (e.g., "delivery boy" sorties by supply ships rendezvous with underway combatants). These undefended ships may require combatant escorts to transit to and visit certain regions, and such escort activities may be within the same area (e.g., IAMD), or necessarily in close company of a combatant (e.g., ASW).
3. The final complication is inclusion of unarmed (or lightly armed) combatants (e.g., landing platform/dock [LPD], landing helicopter dock [LHD], landing helicopter assault [LHA], amphibious, or mine countermeasure [MCM] ships) that may require armed combatant escorts.

In case 1, the mission sets have been worked out ahead of time and the remaining questions are which ships to assign each day to each region to complete as many missions on time as possible. Missions have varying value, and we seek to maximize the total value we can anticipate achieving while satisfying constraints on mobility, simultaneous and conditional mission completion, and varying effectiveness of our combatants and their assignments.

In case 2 we add logistics ships with their own mobility and commodity limitations.

Finally, case 3 can include in the mission set an increased diversity, including such things as an amphibious assault. This involves more ships that need combatant defense, and is most useful for early net assessment.

We anticipate that planners would start with case 1, then refine to case 2, and finally specify the details of case 3 at successively lower-level operational command planning.

DEPLOYMENT NETWORK FLOW MODEL

We introduce a representative scenario called the "Second Battle of Philippine Sea" in Figure 1. (The original WWII battle is well described in Morrison (1963) and the namesake event used as our example by Kline (2010).)

The AOR is partitioned into discrete regions (e.g., see Figure 2).

Each of a set of ships arrives at one of the regions and comes into our control on a known day, not necessarily the first day of the planning horizon, and not necessarily for its duration. We define a node for each day and region. Subsequently, each ship may move forward in time from node to node via adjacent arcs.

There are various types of missions located in regions by day, each with a given value. Some missions are dependent on completion of other prerequisite ones, perhaps on prior days. Such sets of interdependent missions are called mission packages.

Each day, each ship (e.g., see Figure 3) can employ any one of a set of combined mission capabilities (CMCs)—a set of missions a ship can execute simultaneously, perhaps with effectiveness influenced by other missions in the combined mission capability set and the ship's state of readiness (weapon inventories, equipment status, crew training, etc).

The planning problem is to find a set of synchronous paths over space and time for all the ships that maximizes the total anticipated mission value achieved. The following integer linear program, Navy Mission Planner with logistics, seeks the best achievable set of ship deployment schedules. Its introduction reveals the simplifying assumptions made to render this problem tractable while retaining realism. By including logistic requirements in operational mission planning, our goal is to enable concurrent and coordinated planning efforts between operations and logistics.

We create a deployment directed network flow optimization model to find a path for each ship from its original region and day (node) to wherever and whenever the model finds no more missions for it to execute. Because we only know a deployment must end, but not, a priori, where and when the ship deployment should actually best end, all candidate ending regions and days (nodes) are connected by an artificial arc to an ending node. Each pair of adjacent day-region nodes is connected by an arc. We add a supply of one unit of flow to each ship origin, a demand of one unit of flow at its artificial ending node, and require that every node has input flow equal to output flow (either one unit, or none).

Sets and Indices [anticipated cardinality]

- $s \in S$: Ship (hull number and name) or port [$\sim$50]
- $cs \in CS \subseteq S$: Combatant ship (e.g., CG, DDG, LCS) [$\sim$30]

Our scenario is set in a future war at sea conflict where U.S. land forces are stationed in various places in the Philippines and must be resupplied. An adversary threatens the sea lift logistic lines into the Philippines with antiship ballistic missiles; antiship missiles launched from long-range aircraft, submarines, and surface combatants; and torpedo-firing submarines. U.S. Navy ships are needed to protect approaches to strategically located ports of San Diego, Pearl Harbor, Guam, Davao, and Yokosuka from ballistic missile attack, mines, and submarines. U.S. Navy ships also are needed to provide protection to logistics and amphibious lift ships as they make their way across the Philippine Sea. Other ships are tasked as "hunters" to find and destroy enemy surface ships and submarines, or collect critical information needed to defeat the enemy.

The Navy Mission Planner will assist in assigning ships to missions in a two-week tactical scenario as well as provide logistic support schedules. In this scenario, we have received requirements for all the missions noted earlier, and especially to support a convoy of amphibious lift ships during their transit from Guam to Davao, Philippines. Prerequisite missions are identified for a port break out of Guam. For example, a mine warfare (MIW) mission is required a day before the amphibious ships leave port to ensure no mines exist along the exiting channel. An IAMD mission and ASW mission are required outside the port the day of the transit to protect the ships as they depart. An ESCORT mission is needed to protect the group all along its intended track. Likewise, for a "port break in," MIW is needed a day before the group arrives in the Philippines and IAMD and ASW missions mirrored for protection at the terminal port. Constraints to fill these missions include a limited number of combatants available, the need for the missions to be logistically supportable, and the time and distance required of the ships to transit to meet the missions. We begin our tactical scenario mid-war, where ships are already employed to various regions and missions with varying degrees of on-board fuel, stores, and ammunition.

Figure 1. The Second Battle of Philippine Sea. We need a Pacific-wide plan to protect ports and defend a supply convoy from Guam to Davao, Philippines.

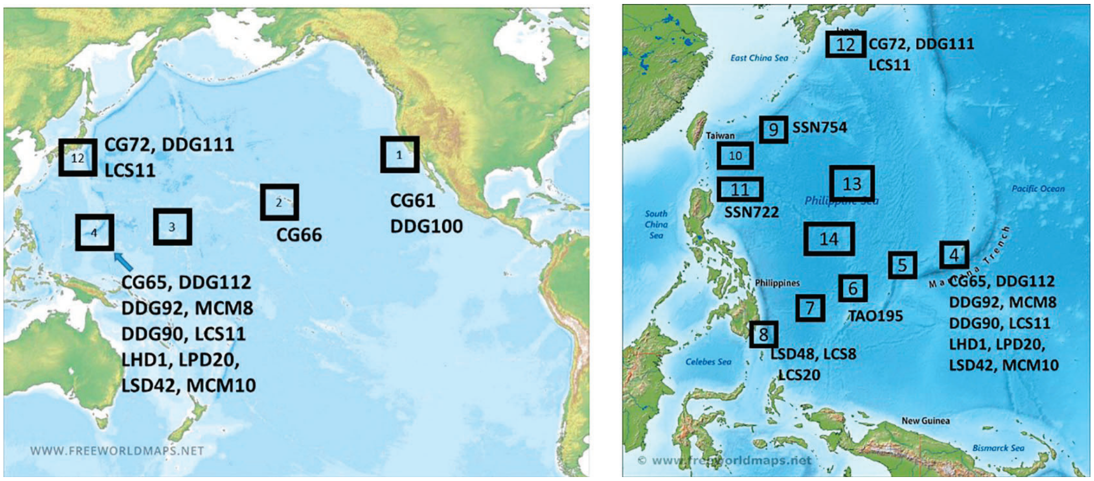

Figure 2. The area of responsibility partitioned into discrete regions. Each region is represented as a node (black box) and a ship can transit between any node pair, as long as she satisfies restrictions on entering either region, such as a combatant escort required for a lightly armed ship. A transit from San Diego (1) to Pearl Harbor (2) is 2,623 nautical miles, employing random 30-degree zig-zag maneuvers underway, requiring five days at 24 knots. We define a node for each ship, day and region it might occupy. Node 8 (Davao, Philippines) is the goal of our resupply efforts. Central coordinates for these regions are in Table A.1 in the appendix.

- $ns \in NS \subseteq S$: Unarmed naval ships (e.g., LHA, LPD, MCM) [$\sim$10]
- $ss \in SS \subseteq S$: Supply ship or port [$\sim$10]
- $ss \in PORT \subseteq SS$: Port [$\sim$5]
- $sx \in SX \equiv CS \cup NS \subseteq S$: Ships that can complete combat missions
- $se \in SE \equiv NS \cup SS \subseteq S$: Ships that may require escorts
- $m \in M$: Mission type (alias m') [$\sim$20] (e.g., ASW, IAMD, NSFS, ESCORT, CLOSE_ESCORT) (see Appendix Table A.2)
- $c \in C_s$: Combined (simultaneous) mission capability (CMC) set for ship s [$\sim$25] (see Table A.5 in the appendix)
- $m \in M_c$: Mission types in combined (simultaneous) mission capability set c (e.g., ship s can simultaneously perform mission types m in CMC set c)
- $r \in R$: Regions in AOR (alias $r1$, $r2$) [$\sim$20]
- $r \in RCS \subseteq R$: Regions navigable by combatant ships (CS)

Figure 3. U.S. Navy guided missile destroyer DDG112 Michael Murphy cost $1 billion, is 155 meters long, displaces nine thousand tons, is crewed by about 320, and is equipped and trained to simultaneously engage in a variety of missions.

$$(CS \bigcap NS \bigcap SS = \theta,\ S = CS \bigcup NS \bigcup SS)$$

$$\boxed{\begin{array}{l} cs \subseteq sx \\ ns = sx \bigcup se \\ ss \subseteq \quad se \end{array}}$$

Figure 4. The three types of entity, combatants *cs,* unarmed naval ships *ns*, and supply ships and ports *ss,* fall in two categories: those that can complete combat missions (*sx*), and those (*se*) that might need combatant escorts.

- $rss \in RSS \subseteq R$: Regions navigable by unarmed naval and supply ships (NS $\cup$ SS), some only if escorted by combatants
- $rloc \in RLOC = RCS \cap RSS \subseteq R$: Regions navigable by all
- $rssx \in RSSX \subseteq R$: Regions navigable by unarmed naval and supply ships (NS $\cup$ SS) only if escorted by a combatant ship
- $rse \in RSE \equiv RSS \backslash RSSX$: Regions always navigable by unarmed ships (NS $\cup$ SS)
- $d \in D$: Days in planning horizon (alias d', d'', $d1$, $d2$) (an ordinal set) [$\sim$15]
- $origin(s,d,r)$: Ship s comes into our control at the start of day d in region r (node)
- $destination(s,d,r)$: Ship s completes any alternate deployment at this node
- $n \in N$: Copy numbers of multiple missions of the same mission type (an ordinal set) [$\sim$5] (e.g., several ships may conduct ASW at the same time within the same region, but with different effectiveness)
- $\{m,d,r,m',d'\} \in MDRMD$: On day d in region r, subsequent mission m can be undertaken only if prerequisite mission m' is fully accomplished on day d'. (See Table A.3 in the appendix, e.g., $\{$IAMD,$d2$,$r3$,ASW,$d1\}$ will require an ASW sweep of area $r3$ on day $d1$, before IAMD can be accomplished the following day.) (*There is no limit on the number of such partial orders among missions, and complex mission packages can be created by combining these*.)
- $\{s,c,m,n,d,r\} \in SCMNDR$: Ship s using combined combat capability set c can perform mission m copy n on day d in region r

Data [units]

- $value_{m,n,d,r}$: Priority (value, effectiveness) of nth mission copy of type m, in region r on day d [value].
- $accomplish_{c,m}$: Anticipated level of accomplishment of combined mission capability set $c \in C_s$, mission $m \in M_c$ [0.0–1.0] (Note that each ship may have its own set of combined mission capability sets, and that some of these sets may contain the same missions, but with different rates of accomplishment to represent the ship choosing to change emphasis between combined simultaneous missions.)

Induced Index Sets

These induced tuple sets are prepared and used to insure consistency of indexing throughout:

- $\{m,n,d,r\} \in MNDR$: 4-tuple exists only if $value_{m,n,d,r} > 0$ or $accomplish_{c,m} > 0$ for some ship that can employ a combined mission capability set that includes mission m on day d in region r, or if mission m in region r on day d is a prerequisite for some other mission.
- $\{m,d,r\} \in MDR$: 3-tuple exists only if $\{m,n,d,r\} \in MNDR$ does for some n.
- $\{s,d\} \in SD$: Ship s in in service during day d (a planner can control this schedule).
- $\{s,d,r\} \in SDR$: Nodes that can be reached by ship s from its starting location (a planner can restrict nodes to exclude a ship on any day from any region, or force occupancy).
- $\{s,c,d\} \in SCD$: Ship s can adopt combined mission capability set c during day d, and $SD\{s,d\}$.

- $\{s,c,d,r\} \in SCDR$: 4-tuple exists only if $\{s,d,r\} \in SDR$ and $c \in C_s$.
- $\{s,c,m,n,d,r\} \in SCMNDR$: 6-tuple exists only if $\{m,n,d,r\} \in MNDR$, and $\{s,c,d,r\} \in SCDR$.
- $\{s,d1,r1,d2,r2\} \in ARCS$: A ship s in region $r1$ on day $d1$ can travel by the beginning of day $d2$ to region $r2$. (The region-to-region steaming distances and ship speeds used to calculate these adjacencies are not shown here.)

In keeping with use in mathematical modeling languages, control of indices is nested, so for example, $\forall\{s,c,d,r\} \in SCDR | m \in \{m,n,d,r\} \in MNDR$ successively controls indices s, c, d, r, and m, given these are in the given domains.

Decision Variables [units]

- $FLOW_{s,d1,r1,d2,r2}$ = 1 if ship s transits from day $d1$ in region $r1$ to start day $d2$ in region $r2$. ($\{s,d1,r1,d2,r2\} \in ARCS$) [binary].
- $CMC_{s,c,d}$ = 1 if ship s adopts combined mission capability set c during period $d(\{s,c,d\} \in SCD)$ [binary].
- $ENGAGE_{s,c,m,n,d,r}$= 1 if ship s engages combined mission capability set c to execute mission m copy n on day d in region $r(\{s,c,m,n,d,r\} \in SCMNDR)$ [binary].
- $U_{m,n,d,r}$: Level of accomplishment by ship s mission m copy n assignment on day d in region $r(\{m,n,d,r\} \in MNDR)$ [0.0–1.0].

Linear Integer Deployment Flow Network Formulation

$$\max \sum_{\{m,n,d,r\} \in MNDR} value_{m,n,d,r} U_{m,n,d,r} \tag{1}$$

$$\text{s.t.} \sum_{\{s,d,r,d2,r2\} \in ARCS} FLOW_{s,d,r,d2,r2,} - \sum_{\{s,d1,r1,d,r\} \in ARCS} FLOW_{s,d1,r1,d,r}$$

$$= \begin{cases} +1 \ origin(s,d,r) \\ 0 \\ -1 \ destination(s,d,r) \end{cases} \quad \forall\{s,d,r\} \in SDR, \tag{2}$$

$$ENGAGE_{s,c,m,n,d,r} \leq \sum_{\{s,d,r,d2,r2\} \in ARCS} FLOW_{s,d,r,d2,r2}$$

$$\forall\{s,c,m,n,d,r\} \in SCMNDR, \tag{3}$$

$$\sum_{\{s,c,d\} \in SCD \setminus c='OOC'} CMC_{s,c,d} = 1 - CMC_{s,'OOC',d} \quad \forall\{s,d\} \in SD, \tag{4}$$

$$\sum_{\{s,c,m,n,d,r\} \in SCMNDR} ENGAGE_{s,c,m,n,d,r} \leq CMC_{s,c,d} \quad \forall\{s,c,d,r\} \in SCDR$$

$$| m \in \{m,n,d,r\} \in MNDR, \tag{5}$$

$$\sum_{\{s,c,m,n-1,d,r\} \in SCMNDR} ENGAGE_{s,c,m,n-1,d,r} \geq \sum_{\{s,c,m,n,d,r\} \in SCMNDR} ENGAGE_{s,c,m,n,d,r}$$

$$\forall s \in S,$$

$$\{m,n,d,r\} \in MNDR | n \geq 2, \tag{6}$$

$$U_{m,n,d,r} \leq \sum_{\{s,c,m,n,d,r\} \in SCMNDR} accomplish_{c,m} ENGAGE_{s,c,m,n,d,r} \qquad \forall \{m,n,d,r\} \in MNDR, \tag{7}$$

$$\sum_{\{s,c,m,n,d,r\} \in SCMNDR} ENGAGE_{s,c,m,n,d,r} \leq 1 \sum_{\{m',n',d',r\} \in MNDR} U_{m',n',d',r} \qquad \forall \{s,c,m,n,d,r\} \in SCMNDR,\ \{m,r,d,m',d'\} \in MRDMD. \tag{8}$$

Decision Variable Domain Restrictions

- $FLOW_{s,d1,r1,d2,r2} \in \{0,1\}$ $\quad \forall \{s,d1,r1,d2,r2) \in ARCS$.
- $CMC_{s,c,d} \in \{0,1\}$ $\quad \forall \{s,c,d\} \in SCD$.
- $ENGAGE_{s,c,m,n,d,r} \in \{0,1\}$ $\quad \forall \{s,c,m,n,d,r\} \in SCMNDR$.
- $U_{m,n,d,r} \in [0,1]$ $\quad \forall \{m,n,d,r\} \in MNDR$.

Discussion

We have taken some liberty to simplify the formulation to convey its intent without clutter induced by, for instance, ships coming in and out of our control over the planning horizon. The objective (1) assesses the anticipated value of (perhaps only partially) completed missions. Each (conservation of flow) constraint (2) equates inflows with outflows for a ship, day, and region. One unit of flow is input as a supply into the network at a ship's origin day-region, and one unit of flow is specified at the ship's (artificial) destination day-region. Constraints (2) form a network for each ship over time with day and origin the ship enters our control with any admissible last day and region connected to a single artificial destination. Each constraint (3) permits a ship to engage missions only in a region on a day the ship occupies that region. Each constraint (4) permits a ship to adopt at most one combined mission capability set, and only if it has not been rendered out of commission (OOC). Each constraint (5) permits missions to be executed in the selected combined mission capability set. Constraints (6) ensure that mission numbers are engaged in ascending order (this is merely for bookkeeping). Each constraint (7) accounts for the level of achievement of a mission by all ships committed to it. Each constraint (8) ensures that a mission will not be engaged until a prerequisite mission has been completed.

We note that it is possible for this model to advise partial achievement of a high-value mission, rather than complete achievement of lower-value ones. This is a feature, not a bug. We can instrument our model to preclude such behavior, but we don't want to do that. We believe this offers the naval planner insight in ship resource decisions and mission prioritization, and may lead to reprioritization during the planning process if deemed necessary.

ADDITIONAL FEATURES FOR LOGISTICS SUPPORT AND ESCORTS

We introduce logistics and defensive escort features to our operational planner.

Sets and Indices for Logistics and Escorts [cardinality]

- $i \in I$: Commodity category (e.g., DFM ship fuel, JP5 aviation fuel, dry stores [STOR], and ORDN ordinance [ORDN]) [4]

Data for Logistics and Escorts [units]

- $cap_{s,i}$: Capacity of ship s for commodity category i [i-units]
- $init\_load_{s,i}$: Initial load of ship s, commodity i [fraction of $cap_{s,i}$]

- $use_{s,c,i}$: Daily consumption of commodity i by Navy ship s employing combined mission capability set c. [i-units]
- $safety_i$: Safety stock fraction of capacity for commodity i [fraction]
- $extremis_i$: Extremis stock fraction of cargo category i [fraction] $\left(0 < extremis_i < safety_i < 1\right)$
- $reward_i$: Reward per unit of inventory above safety stock level [value]
- pen_safe_i: Penalty per unit of violation of safety stock for commodity i [value]
- pen_extr_i: Penalty per unit violation of extremis stock for commodity i [value]
- pen_out_i: Penalty per unit violation below zero stock for commodity i [value] $\left(pen\_out_i > pen\_ext_i > pen\_safe_i > 0\right)$
- pen_escort: Penalty per ship-day shortage of escorts. [value/escort-day]
- $force\_ratio_r$: Ratio of armed combatants to other escorted ships in region r [ratio]
- $max\_reps\_per\_day_s$: Maximum number of daily replenishment events for ship s
- $min\_days\_between\_reps$: Minimum number of days separating replenishment events
- $max\_days\_between\_reps_s$: Maximum number of days separating replenishment events for ship s
- $convoy_{s,d}$ = 0 none; otherwise a convoy number for ship s to join on day d

Binary Model Option Toggles

- *supply*: Feature required for logistics support
- *disable*: Disable any ship that runs out of some commodity.
- *escort*: Restrict unarmed ships from entering an RSSX region without some combatant escort also in that region.
- *close*: Restrict unarmed ships from entering an RSSX region without some combatant escort in close accompaniment there.
- *armed*: Require in any day in any region that each unarmed ship there be escorted by some number of armed escorts.

Induced Index Sets

- $\{s,d,i\} \in SDI$: Ship s may use commodity i on day d.
- $\{ss,sx,d,r,i\} \in SSDRI$: Supply ship ss can be collocated with combatant or noncombatant customer ship sx on day d and can transfer commodity i.
- $\{s,s1,d1,r1,d2,r2\} \in PAIRS$: ARCS that can be occupied by ships in convoy, i.e., $convoy_{s,d} > 0 \wedge convoy_{s,d} = convoy_{s1,d} \wedge \{s,d1,r1,d2,r2\} \in ARCS \wedge \{s1,d1,r1,d2,r2\} \in ARCS$ (this is notational shorthand for the actual editing of ARCS to remove those rendered inadmissible due to joint convoy membership of ships s and $s1$).

Decision Variables for Logistics and Escorts [units]

- $XFER_{ss,sx,d,r,i}$: Volume of commodity i transferred from supply ship ss to ship sx on day d in region $r(\{ss,sx,d,r,i\} \in SSDRI)$ [i-units]
- $SLACK_{s,d,i}$: Ship s, at end of day d, commodity i stock in excess of safety stock $(\{s,d,i\} \in SDI)$ [i-units]
- $V\_SAFE_{s,d,i}$: Violation of safety stock level for ship s, day d, commodity $i(\{s,d,i\} \in SDI)$ [i-units]
- $V\_EXTR_{s,d,i}$: Violation of extremis stock level for ship s, day d, commodity $i(\{s,d,i\} \in SDI)$ [i-units]

- $V\_OUT_{s,d,i}$: Violation of positive stock level for ship s, day d, commodity $i(\{s,d,i\} \in SDI)$ [i-units]
- $ESCORTS\_SHORT_{d,r}$: Insufficient armed escorts when and where required by *armed* option [escorts]

Additional Logistics and Escort Formulation

$$\begin{aligned}
&\Bigg[+\sum_{\{s,d,i\}\in SDI} \big(reward_i/cap_{s,i}\big) SLACK_{s,d,i} \\
&-\sum_{\{s,d,i\}\in SDI} \big(pen\_safe_i/cap_{s,i}\big) V\_SAFE_{s,d,i} \\
&-\sum_{\{s,d,i\}\in SDI} \big(pen\_extr_i/cap_{s,i}\big) V\_EXTR_{s,d,i} \\
&-\sum_{\{s,d,i\}\in SDI} \big(pen\_out_i/cap_{s,i}\big) V\_OUT_{s,d,i}\Bigg] \big| supply \\
&-pen\_escort \sum_{d\in D, r\in R} ESCORTS\_SHORT_{d,r} \big| armed, \qquad \text{(9) (add to (1))}
\end{aligned}$$

$$\begin{aligned}
&init\_load_{s,i} cap_{s,i} \\
&\quad + \sum_{\substack{\{s',s,d',r,i\} \in SSDRI \\ |d' \leq d}} XFER_{s',s,d',r,i} - \sum_{\substack{\{s,s',d',r,i\} \in SSDRI \\ |d' \leq d}} XFER_{s,s',d',r,i} \\
&\quad - \sum_{\substack{\{s,c,d\} \in SCD, \\ \{s,c,d'\} \in SCD|,d' \leq d}} \big(use_{s,c,i} - use_{s,'TRANSIT',i}\big) \sum_{\{s,c,m,n,d',r\}\in SCMNDR} ENGAGE_{s,c,m,n,d',r} \\
&\quad - \sum_{\substack{\{s,c,d\} \in SCD, \\ \{s,c,d'\} \in SCD|,d' \leq d}} \big(use_{s,'TRANSIT',i}\big) \\
&\quad = safety_i cap_{s,i} + SLACK_{s,d,i} - V\_SAFE_{s,d,i} - V\_EXTR_{s,d,i} - V\_OUT_{s,d,i} \\
&\qquad\qquad \forall\{s.d,i\} \in SDI|supply, \qquad (10)
\end{aligned}$$

$$\begin{aligned}
&\sum_{\{se,d,rssx,d2,r2\}\in ARCS} FLOW_{se,d,rssx,d2,r2} \\
&\quad \leq \sum_{\{cs,c,'ESCORT',n,d,rssx\}\in SCMNDR} ENGAGE_{cs,c,'ESCORT',n,d,rssx} \\
&\qquad\qquad \forall rssx \in RSSX, \\
&\qquad\qquad \{se,d,rssx\} \in SDR|escort, \qquad (11)
\end{aligned}$$

$$\begin{aligned}
&\sum_{\{se,d,rssx,d2,r2\}\in ARCS} FLOW_{se,d,rssx,d2,r2} \\
&\quad \leq \sum_{\{cs,c,'ESCORT',n,d,rssx\}\in SCMNDR} ENGAGE_{cs,c,'ESCORT',n,d,rssx} \\
&\qquad\qquad \forall rssx \in RSSX, d \in D|close, \qquad (12)
\end{aligned}$$

$$\sum_{\{ss,s,d,r,i\}\in SSDRI} XFER_{ss,s,d,r,i} \le cap_{ss,i}\left(\sum_{\{ss,c,'INREP',n,d,r\}\in SCMNDR} ENGAGE_{ss,c,'INREP'n,d,r} + \sum_{\{ss,c,'UNREP',n,d,r\}\in SCMNDR} ENGAGE_{ss,c,'UNREP'n,d,r}\right) \quad \forall\{ss,d,r\}\in SDR, i\in I|supply, \tag{13}$$

$$\sum_{\{ss,s,d,r,i\}\in SSDRI} XFER_{ss,s,d,r,i} \le cap_{s,i}\left(\sum_{\{s,c,'INREP',n,d,r\}\in SCMNDR} ENGAGE_{s,c,'INREP',n,d,r} + \sum_{\{s,c,'UNREP',n,d,r\}\in SCMNDR} ENGAGE_{s,c,'UNREP',n,d,r}\right) \quad \forall\{s,d,r\}\in SDR, i\in I|supply, \tag{14}$$

$$CMC_{s,'OOC',d} \ge V\_OUT_{s,d,i} / \max\left(1, -init\_load_{s,i}cap_{s,i} + \sum_{\substack{\{s,d'\}\in SD \mid d'\ge d\\ \wedge d'-d < max\_days\_between\_reps_S}} \max_{c\in C}\{use_{s,c,i}\}\right) \quad \forall\{s,d,i\}\in SDI|disable, \tag{15}$$

$$CMC_{s,'OOC',d} \le V\_OUT_{s,d,i} \forall\{s,d,i\}\in SDI|disable, \tag{16}$$

$$\sum_{\substack{\{s,c,m,n,d,r\}\in SCMNDR\\ |m='INREP'\vee m='UNREP'}} ENGAGE_{s,c,m,n,d,r} \le max\_reps\_per\_day_s \quad \forall\{s,d\}\in SD|supply, \tag{17}$$

$$\sum_{\substack{\{s,c,m,n,d',r\}\in SCMNDR\\ |(m='INREP'\vee m='UNREP')\\ |\wedge d'\ge d\wedge d'-d < min\_days\_between\_reps}} ENGAGE_{s,c,m,n,d',r} \le 1 \quad \forall\{s,d\}\in SD \; |d < |D| - min\_days\_between\_reps \; \wedge supply, \tag{18}$$

$$\sum_{\substack{\{s,c,m,n,d',r\}\in SCMNDR\\ |(m='INREP'\vee m='UNREP')\\ |\wedge d'\ge d\wedge d'-d < max\_days\_between\_reps_S}} ENGAGE_{s,c,m,n,d',r} \ge 1 \quad \forall\{s,d\}\in SD \; |d < |D| - max\_days\_between\_reps_s \wedge supply, \tag{19}$$

$$\sum_{\{cs,d,r,d2,r2\}\in ARCS} FLOW_{cs,d,r,d2,r2} \ge force\_ratio_r \sum_{\{se,d,r,d2,r2\}\in ARCS} FLOW_{se,d,r,d2,r2} \quad \forall r\in R, d\in D|armed, \tag{20}$$

$$FLOW_{s,d1,r1,d2,r2} = FLOW_{s1,d1,r1,d2,r2} \quad \forall\{s,s1,d1,r1,d2,r2\}\in PAIRS. \tag{21}$$

Logistics and Escort Decision Variable Domains

- $XFER_{ss,sx,d,r,i} \in \left[0,\min\left(cap_{ss,i},cap_{sx,i}\right)\right] \quad \forall\{ss,sx,d,r,i\}\in SSDRI$
- $SLACK_{s,d,i} \in \left[0,\left(1-safe_i\right)cap_{s,i}\right] \quad \forall\{s,d,i\}\in SDI$

- $V\_SAFE_{s,d,i} \in [0,(safe_i - extremis_i)cap_{s,i}] \quad \forall\{s,d,i\} \in SDI$
- $V\_EXTR_{s,d,i} \in [0,extremis_i cap_{x,i}] \quad \forall\{s,d,i\} \in SDI$
- $V\_OUT_{s,d,i} \geq 0 \quad \forall\{s,d,i\} \in SDI$
- $ESCORTS\_SHORT_{d,r} \geq 0 \quad \forall d \in D, r \in R$

Discussion for Logistics and Escorts

Now that we are accounting for commodity consumption and resupply, the objective (1) also rewards for exceeding inventory safety stock levels and assesses penalties for falling below safety stock, or below (lower) extremis stock, or even for running out. (In practice, running out is avoided by slowing down to a more fuel-efficient speed.) Each constraint (10) accounts for a commodity stock level on a ship at the end of a day. This constraint computes daily stock levels by summing over the ship's active days; this relatively involved computation yields a model easier to solve than one with conventional daily inventory variables. Each constraint (11) permits a ship to enter a region where it requires an escort only on a day for which some combatant has engaged the ESCORT mission there; if the ESCORT mission has been engaged in a region, any number of supply ships may enter there. If CLOSE_ESCORT is required, each constraint (12) requires that there will be at least one combatant per escorted ship. Each pair of constraints (13) and (14) governs volume transferred between ships collocated in a region on a day. Each pair of constraints (15) and (16) disable a ship that runs out of fuel (see constraint (4)). Each constraint (17) limits the maximum daily number of replenishment events for a ship. Constraints (18) limit the frequency with which a ship can be interrupted by a replenishment. Constraints (19) define an epoch within which a ship must be replenished. Each constraint (20) requires for each region on each day at least a given ratio of combat ships per defended ship. Constraints (21) synchronize flows between ships engaged in the same convoy.

REVISIONS AND PERSISTENCE

Navy mission planning is like three-dimensional chess: We are moving discrete ships (our chess pieces) from region to region (positions on our chessboard) and time (layers of chessboards) to complete particular space-time missions over a finite planning horizon, with side constraints on how and where our ships can move, and among the missions we can complete.

Such planning may be for purposes of assessment. We might ask questions such as:

- Can we complete this mission set over this time horizon with these ships?
- If another ship can be included, how does this influence our plan?
- If we lose a port, a ship, or a ship is delayed in arrival to the AOR, what influence does this have on our plan?
- Can we, or do we need to modify the mission schedule?
- What missions cannot be accomplished?

Many of these questions will arise as a plan is prepared in advance of anticipated execution.

This prototypic research is also intended to provide optimization-based decision support to operational commanders when bad things happen during plan execution: *"No plan survives initial contact with the enemy."* (paraphrased from von Moltke, 1871)

Although optimization-based decision support can effectively solve complicated problems, it does have its limitations: "Most optimization-based decision support systems are used repeatedly with only modest changes to input data from scenario to scenario. Unfortunately, optimization (mathematical programming) has a well-deserved reputation for amplifying small input changes into drastically different solutions. A previously optimal solution, or a slight variation of one, may still be nearly optimal in a new scenario and managerially preferable to a dramatically different solution that is mathematically optimal" (Brown et al., 1997).

If we are deep in analysis of a plan under development, excessive numbers of revisions responding to slight refinements are an annoying distraction. Most planners would prefer to keep the parts they like, and merely improve those they don't.

If we are revising a plan already in execution, our ships have already been given deployment orders, perhaps loaded commodities in anticipation of carrying out certain missions, and may be underway or in engagement. The last thing we want is unnecessary turbulence leading to excessive messaging and confusion.

There are a number of ways to mitigate unnecessary changes and reduce turbulence between plan revisions, so-called "persistence" features in an optimization model.

The most severe persistent restriction is fixing an employment for a ship, region, time period, and mission. If a planning system is being used over time, such fixing will be necessary as the near-term future becomes the present and past as we progress. If we are revising a future plan, we might find some actions so attractive and dominant they merit fixing. For such future planning, moderation is a virtue: the last thing we want is to hobble our forces due to some unintended myopic restriction, or, worse, unintentionally render plans infeasible by inadmissible restrictions (e.g., requiring an underway ship speed of 200 knots is inadmissible).

Although there are a number of techniques to introduce persistence (Brown et al., 1997), for purposes of illustration we will focus on assignment of a ship to a region, perhaps during a particular day, and perhaps for a particular mission. Assignments of other sorts of activities proceed in an analogous fashion.

Additional Data [units]

- $\hat{legacy}_{s,r,d,m} \in \{0,1\}$: Legacy plan to send ship s to region r during day d for mission m [binary]
- $persistence\_penalty$: Cost to change an assignment to legacy plan [value]
- fog: Daily penalty discount rate ("fog of war", e.g., 0.1, or 10% per day)

Additional Formulation Terms [units]

To fix legacy terms, add:

$$\sum_{\{s,c,m,n,d,r\}\in SCMNDR} ENGAGE_{s,c,m,n,d,r} = \hat{legacy}_{s,r,d,m} \ \forall\{s,r,d,m\} \in \hat{legacy}_{s,r,d,m}. \tag{22}$$

Hallmann (2009) uses this expediency to fix the location of supply ships for some time periods, creating "gas stations" for combatants.

A slightly less drastic restriction is to sense and penalize changes to a legacy schedule. To merely penalize solutions that do not follow legacy terms, add:

$$+persistence\_penalty\left(\sum_{\{s,c,m,n,d,r\}\in SCMNDR|\hat{legacy}_{s,r,d,m}=1} e^{-fog\ d}(1-ENGAGE_{s,c,m,n,d,r})\right.$$
$$\left.+\sum_{\{s,c,m,n,d,r\}\in SCMNDR|\hat{legacy}_{s,r,d,m}=0} e^{-fog\ d}(ENGAGE_{s,c,m,n,d,r})\right).$$

Restrictions of index tuple subsets $\hat{legacy}_{s,r,d,-}$ or $\hat{legacy}_{s,r,-,-}$ can be accommodated similarly.

Verbal Discussion

There are two levels of severity here. Each constraint (22) restricts an employment fixed by the planner. The two new terms in the objective function (1) encourage, rather than fix, following a legacy plan by respectively assessing and penalizing any additions to or deletions from the legacy plan. The discount rate encourages delaying changes into the future when possible.

Using mechanisms like this, we can shape any revision to our liking. *The caution here is not to set persistence penalties so high as to end up "steering by our wake," sticking with legacy plans to the detriment of otherwise attractive revisions.*

We have resorted to a two-phase solution that works well. First, we solve for combatant plans only. We then pre-seed these binary variable values and employ the persistence feature to use these as a guide when adding logistics and escort embellishments. *An experienced manual planner would proceed exactly this way, first assembling a combatant plan greedy for mission value achievement, then observing logistic infeasibilities and injecting logistics forces and perhaps assigning escorts for these.*

DEPLOYMENT PATH SELECTION MODEL

An alternate means to solve our problem is to enumerate all paths for each ship through our directed deployment flow network, and select a path for each ship that, in concert with all other paths selected, satisfies constraints and leads to completing the maximum total value of missions achieved.

Our directed deployment flow network for each ship is, by construction, acyclic and topological. Thus, we can easily enumerate how many paths there are (in polynomial time, see Figure 5).

Additional Binary Model Toggles

- *solve_flows* or *solve_paths* direct which model to use.

Additional Sets and Indices for Path Formulation [cardinality]

- $p \in P$: Employment schedules, alias p' [exponential]
- $p \in P_s \subseteq P$: Employment schedules for Navy ship s [exponential] ($\underset{s}{\cup} P_s \equiv P$, P_s is a partition of P)
- $\{d,r\}_p$: Employment path p on day d visits region r

Additional Decision Variables

- $Y_p = 1$ to select path p, 0 otherwise [binary]

Deployment Path Selection Optimization Model

Using the directed deployment network flow model, replace constraints (2) by:

$$\sum_{p \in P_s} Y_p \leq 1 \quad \forall s \in S \tag{24}$$

$$\sum_{\{s,c,d\} \in SCD} CMC_{s,c,d} = \sum_{p \in P_s, \{d,r\}_p} Y_p \quad \forall \{s,d,r\} \in SDR \tag{25}$$

```
for n = 1 to NODES: # nodes in lexicographic sequence
        count(n) = 0;
count(1) = 1;
for n = 1 to NODES:
        for j in FORWARD_STAR(n):  # nodes adjacent to n
            count(j) = count(j) + count(n);
print "count(NODES)"
```

Figure 5. Abstract algorithm to count all directed paths in a lexicographic network such as the deployment flow network for each ship. ("Forward star" is defined in, e.g., Ahuja et al. (1993).)

Replace each subsequent appearance of a *FLOW* variable with an equivalent expression using *Y* and replace domain restrictions on *FLOW* by:

$$Y_p \in \{0,1\} \quad \forall p \in P$$

One can replace the exponential number of all directed paths by a sampled subset and achieve a restricted model with objective value no better, and perhaps not much worse. This technique has been used before by Dugan (2007), Hallmann (2009), Silva (2009), Brown et al. (2013), and most recently by Baker (2019). The representative example we will solve with our flow model in this paper has 1,058,826,559,993 paths. Baker compares a flow deployment model with a depth-first searched sample, and a randomly generated one, each with about 200 thousand paths. Using his flow results as an upper bound of the mission value objective, he finds: "*Sampling only about 0.000019% of paths for both truncated brute-force enumeration achieves 90% mission value and random path generation achieves over 93% of mission value*."

There is an additional charm of directed paths: each path can easily be filtered by (e.g., nonseparable, nonlinear) properties of all its nodes in the sequence they are traversed. After inadmissible paths are removed, the remaining optimization model is still a linear integer program in terms of the remaining paths.

IMPLEMENTATION

Navy Mission Planner data and displays are managed with Microsoft Excel and Visual Basic for Applications (VBA) (2020). The optimization models have been implemented in both GAMS (2020) and Python-Pyomo (2020), with alternate solvers CPLEX (2020) and Forrest and Lougee-Heimer (2005). The combination of Excel, VBA, Python, Pyomo, and CBC includes components that should be admissible for classified computer systems either because (in the case of Excel and VBA) they are standard, or (in the case of Python, Pyomo and CBC) they are open -source and can be scanned for forbidden functionality.

The illustrative scenarios share 24 entities (14 combatant ships, six naval ships, as many as four supply ships, and four ports for replenishments, see Appendix Table A.4), a 14-day planning horizon with 231 missions of 15 types and 12 prerequisite relationships in mission packages. There are a number of scenario cases: (A) scheduling combatant and naval ship mission plans only; (B) adding logistic support from CLF shuttle supply ships and ports; (C) replanning after the loss of a combatant; (D) replanning when a port is closed for days; and (E) stationing supply ships near ports.

Case A ignores logistics and assigns combatants and naval ships to engage as many missions as time and geography allow. This provides a greedy good case, albeit ships requiring escorts in some regions cannot access them. Table 1 shows this case assigns ships to missions achieving almost 93% of total mission value. This case reveals two missions that no ship has time to reach and engage, so the best we can achieve is 99%.

After observing the deployments and mission accomplishments of case A, we planners worry about fuel in the Western Pacific, and the lack of flexibility that can accrue from shortages. Initial fuel inventory levels are all above 80% of capacity, with safety stock 50%, and extremis 25%. A CG with full fuel can travel at 24 knots for about 11 days. Other combatants have comparable unsupplied endurance. Case B assigns a supply ship (TAO195 Leroy Grummand) west of Guam in region r6, defended by a Littoral Combat Ship (LCS8 Montgomery). This "gas station" achieves an inconsequential increase in mission value to 93.1%, but more importantly keeps our ships replenished with more fuel in case of contingencies and readies them for a follow-on 14-day planning horizon. Use of persistence guidance from case A significantly reduces solution time. These results show us the benefits of including logistics in planning.

Case C follows case B (and its persistent guidance), but takes cruiser CG61 Monterey out of service at the beginning of day *d*5 due to battle damage. This is a grave loss. She was expected to achieve over 7.5% of all planned mission value over our entire planning horizon, just from day *d*5

Table 1. Representative scenario solution statistics.

Case	Scenario	Constraints	Continuous variables	Binary variables	Persistence with	Mission value %
A	No supply ship	33K	7K	27K		93.0
B	With a supply ship	123K	10K	61K	A	93.1
C	CG61 lost *d*5	121K	9K	59K	B	93.5
D	Guam out *d*4–*d*9	33K	7K	26K	B	86.8
E	Supply ships near ports	1,134K use (11)	11K	65K	B	95.7

Notes: This sample problem has a 14-day planning horizon over 12 regions served by four supply ports. There are 14 combatants, six unarmed Navy ships, and as many as four Combat Logistics Force supply ships. The only fixed ship day-regions are their initial arrivals. Case C has 121K constraints, 9K continuous variables, 59K binary variables, uses persistence guidance from case B and achieves total mission value of 93.5% of total mission value. Each case is run for at most 60 minutes on a portable workstation (Intel Core I7-8750H @ 2.2 GHz-6 cores PassMark (2019) average 12,483). Typical integrality gap (the difference between the best solution found and a bound on how much better a solution might be) is 1%; this is a concern when comparing results between scenarios.

on. Fixing all prior-planned activities through the end of *d*4, and then replanning to recover subsequent mission value by shuffling our ships among regions and missions, we can reduce this loss to 3.4%. To minimize turbulence in our revision, we also use the persistence feature with the legacy case B to discourage frivolous changes. Despite this, there are 98 changes of mission assignments from case B, demonstrating at once the difficulty of revisions and the flexibility of our combatants. If we want to reduce the number of revisions (that requires a lot of message traffic with our ships), by gradually increasing *persistence_penalty* the number of changes can be reduced to as few as 24. This costs some loss of mission value accomplishment, but might appeal to a planner.

Case D restores CG61 and takes Guam out of service during days *d*4–*d*9 for mine clearance operations. LSD48 Ashland is stranded pier side in Guam for the duration of this port closure. Fixing prior-planned activities through day *d*4, closing Guam and disabling LSD48 for this epoch, and replanning achieves mission value 86.8%.

Recognizing the value and vulnerability of our ports, case E uses the conditions of case B and stations a TAO supply ship at sea adjacent to each port. This raises mission value achieved to 95.7%. The advantage here is that combatants defending each port no longer have to leave station to enter the port to refuel, and thus can provide continuous defense.

Each scenario solve creates a file that can be modified and read later by the optional persistent model feature. A subsequent run with the toggle *use_persistence* searches for and reads this file, ignoring any entries not recognized and creating persistent features for those it does. This file includes each suggested entity: ship, region, day, and mission with direction to either *TRY_0*, *TRY_1*, *FIX_0*, or *FIX_1*. The TRY alternatives are used to build a preferred legacy solution $\widehat{legacy}$ penalty for the objective (1) and the *FIX_0* and *FIX_1* options create constraints (22).

Clean-sheet scenarios are not easy to solve. There are a number of causes, one of which is symmetry among identical ships that confounds integer enumeration. Discovering an incumbent with an integrality gap of 5% or less can take 30 minutes. Attempting for a 1% gap takes longer than an hour, sometimes much longer. After a lot of experimentation, we have set the maximum compute time at 60 minutes and would only press further if there were some important ambiguity.

In practice, we would not tolerate longer solve times. In the interests of gaining quick, early insight, the planner(s) will use something like case A to get a rough assessment of the capability of our intended force to engage with the planned mission schedule. During these early exercises, the combined mission capability sets may be massaged, and ships added or dropped from our force. The relative values of missions may be adjusted to re-direct attention. The prerequisite mission packages present a key complication that is better understood and appreciated once planners see how the optimization pieces these together. Once there is reasonable alignment of our forces with missions, the added features discussed here can be introduced to flesh out a logistically feasible plan.

INNOVATIONS AND PRACTICAL OPERATIONAL PLANNING

Although every Navy surface warfare officer knows that a combatant ship can engage in multiple, simultaneous missions, we don't think anybody has suggested anything resembling the CMCs we introduce, and recommend pursuing these with further analysis. Existing sea trials and qualitative or even quantitative evaluation are welcome, and the Navy has plenty of data to support such. In addition, no automated decision tool we know of considers prerequisite and concurrent required missions. These are normally handled manually by the operational planning staff.

Our recommendation for a complicated cumulative computation of inventories in constraints (10), in lieu of a conventional textbook recursion computing an inventory level from period-to-period, derives from theory and empirical experience. This just solves much faster. For a deeper dive, consider the network structure that (10) spans, and examine the description of underlying bases in Brown et al. (1977).

The path enumeration equivalence to the network flow model suggests some alternate means to approximately solve very large planning problems. Each path presents a complete ship deployment schedule that can be evaluated in more detail than the equivalent separable linear flow.

The operational planning we have seen needs quantitative support. With manual planning, mistakes happen. We have witnessed an operational planning session where a junior officer informed a superior, "Sir, substituting that ship for that mission would require it to make 100 knots."

We also prefer to have documented, quantitative grounds to support our plans: "Here is what we have assumed, here is what we recommend, and here is why."

Integrating operational and logistics planning is not simple, but can render much superior operational plans, and much sooner than the conventional iteration between operational planners and logisticians, going back and forth, seeking some feasible middle ground. We prefer to characterize the feasible middle ground, and get the best solution there in one unified planning exercise.

Most important, persistence features are absolutely essential to polishing an operational plan. As planning progresses, some actions can be fixed whereas others might be attractive, but not so much so as to restrict any alternative. We can expect scores of scenarios to be run. The bonus surprise is that these persistence penalties greatly accelerate solution times, and our planning time is a key planning constraint. We have experimented, and enjoyed a couple orders of magnitude improvement in solution times to achieve any desired solution resolution.

Persistence can also be employed to reduce the number of changes to an ongoing operation to deal with surprises.

CONCLUSION

Navy Mission Planner has been used for campaign analyses often enough that we have become comfortable it represents about the right fidelity to suggest operational mission assignment plans (i.e., we no longer get complaints about the difficulty of representing some essential feature). The most important innovations are (1) combined mission capabilities than can express multimission, ship-specific efficiencies, (2) mission packages expressing dependencies among missions, and (3) explicit inclusion of naval logistics. This is not the first time we have demonstrated and promoted the charm of persistent modeling to aid revising and polishing a plan (see, e.g., Brown et al. 1996). Here, this qualitatively improves solve times, as well.

This planner can scale up for many more ships, missions, and a longer planning horizon. However, a longer planning horizon would be rather fanciful. At some point, an operational planning horizon becomes strategic; although our Navy prides itself on its ability to maintain long deployments at sea, the sorts of scenarios we address would, if extended over time, likely require our ships to withdraw to receive pier-side attention. This invites a different sort of planning.

Traditionally, combatant plans have been developed independently by operational planners, then sent to logisticians. The logisticians do their best, and send plans back to operational planners. It is in this spirit that the Replenishment at Sea Planner (Brown et al., 2017) in daily worldwide use by our Navy receives operational plans and has little influence on their formulation. We have witnessed this iterative communication, and suggest a better way to integrate planning to achieve a better result than with restricted iterations for a limited number of iterations. Our overarching goal has been to promote coordinated operational planning.

The U.S. Navy is adopting new semi- and fully autonomous platforms: ships, aircraft, and submersibles. Navy Mission Planner appeals for organizing joint operations between and among all deployed platforms.

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Rick Rosenthal introduced a number of notions we adopt here as our own and Matt Carlyle has contributed continuing advice. For appreciation of the art of naval warfare, we refer the reader to Hughes and Girrier (2019). It has been a pleasure to develop Navy Mission Planner in concert with junior naval officer students who are expected to execute such plans at sea.

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APPENDIX. DATA MODEL

Tables A.1–A.6 show how key data elements are represented as worksheets in Excel. Not shown are additional worksheets with routine model controls, geolocations of and distances between regions, ship speeds and commodity consumption rates, as well as region-wise restrictions on and escort requirements for access of ship classes.

Table A.1. Region center coordinates, with indication of regions unarmed ships can enter without escort (RSS), and with (RSSX).

Region	LON	LAT	RSS	RSSX
San_Diego	117.15	32.69	x	
*r*1	117.5	32.5	x	
*r*2	155.25	19.0	x	
Pearl	157.97	21.34	x	
*r*3	−155.6	14.0	x	
*r*4	−144.5	13.5	x	
Guam	−144.65	13.46	x	
*r*5	−139.3	11.2	x	
*r*6	−135.0	10.0	x	
*r*7	−131.2	06.0		x
*r*8	−126.5	06.0		
*r*9	−129.2	24.0		
*r*10	−123.6	22.3		
*r*11	−125.1	18.6		
*r*12	−135.5	32.2	x	
*r*13	−132.6	19.1		
*r*14	−130.3	15.2		
Yokosuka	−139.66	35.28	x	

Table A.2. Missions types.

Type	Description
IAMD	Integrated air and missile defense
ASW	Antisubmarine warfare
NSFS	Naval surface fire support
MIO	Maritime interdiction operations
LOG	Logistics mission between naval ships
INTEL	Intelligence collection
LIFT	Amphibious lift (assault)
MIW	Mine warfare
STRIKE	Strike operation
SUW	Surface-to-surface strike
INREP	In-port replenishment
UNREP	Underway replenishment
ESCORT	Provide
TRANSIT	Transit between mission regions

Table A.3. Sample mission packages from an operational plan.

			Prerequisite	
Mission	Region	Day	Mission	Day
ESCORT	*r*4	*d*3	MIW	*d*2
		*d*4	MIW	*d*3
LIFT	*r*4	*d*3	MIW	*d*2
		*d*4	MIW	*d*3
MIW	*r*4	*d*1	ASW	*d*1
		*d*2	ASW	*d*2
		*d*3	ASW	*d*3
MIW	*r*4	*d*1	IAMD	*d*1
		*d*2	IAMD	*d*2
		*d*3	IAMD	*d*3

Notes: Each of these missions, by mission type, region, and day, has a number of required prerequisite missions in that region. For example, before ESCORT can be engaged in region *r*4 on day *d*3, MIW sweep must have been conducted on the prior day. MIW, in turn, has its own simultaneous requirements for ASW and IAMD missions. The package shown here is preparing for a "break out" departure from Guam on day *d*4.

Table A.4. Entities.

Hull	Ship	Class	Type	SX?	SE?	Start	Kts	CMCs
CG61	Monterey	CG	CS	x		*d*1 *r*1	24	C2, C4
CG65	Chosin	CG	CS	x		*d*1 *r*4	24	C1, C2, C3, C4
CG66	Hue City	CG	CS	x		*d*1 *r*2	24	C1, C4
CG72	Vella Gulf	CG	CS	x		*d*1 *r*12	24	C3, C4
DDG90	Chung-Hoon	DDG	CS	x		*d*1GU	24	C5, C6, C7, C8
DDG92	Momsen	DDG	CS	x		*d*1GU	24	C5, C6, C7, C8
DDG100	Kidd	DDG	CS	x		*d*1SD	24	C5, C6, C7, C8
DDG111	Spruance	DDG	CS	x		*d*1YO	24	C6, C7
DDG112	Michael Murphy	DDG	CS	x		*d*1GU	24	C6, C7
LCS8	Montgomery	LCS	CS	x		*d*1 *r*8	24	C10, C11, C12, C13
LCS11	Sioux City	LCS	CS	x		*d*1 *r*12	15	C10, C11, C12, C13
LCS20	Cincinnati	LCS	CS	x		*d*1 *r*8	15	C10, C11, C12, C13
SSN722	Key West	SSN	CS	x		*d*1 *r*11	24	C19, C20
SSN754	Topeka	SSN	CS	x		*d*1 *r*9	24	C19, C20
MCM8	Pioneer	MCM	NS	x	x	*d*1 *r*4	24	C18
MCM10	Warrior	MCM	NS	x	x	*d*1 *r*4	24	C18
LHD1	Wasp	LHD	NS	x	x	*d*1GU	24	C15, C16, C17
LPD20	Green Bay	LPD	NS	x	x	*d*1GU	15	C15, C16, C17
LSD42	Germantown	LSD	NS	x	x	*d*1GU	15	C15, C16, C17
LSD48	Ashland	LSD	NS	x	x	*d*1 *r*8	15	C15, C16, C17
TAO195	Leroy Grummand	TAO	SS		x	*d*1 *r*6	24	UNREP
TAO196	Kanawha	TAO	SS		x	*d*1 *r*1	24	UNREP
TAO197	Pecos	TAO	SS		x	*d*1 *r*2	24	UNREP
TAO199	Tippecanoe	TAO	SS		x	*d*1 *r*12	24	UNREP
SD	San Diego	PORT	SS		x	*d*1SD	0	INREP
PH	Pearl Harbor	PORT	SS		x	*d*1PH	0	INREP
YO	Yokosuka	PORT	SS		x	*d*1YO	0	INREP
GU	Guam	PORT	SS		x	*d*1GU	0	INREP

Notes: Guided missile cruiser CG61 Monterey is a combatant ship, is available starting on day *d*1 in region *r*1, and can operate in one of the combined mission capability sets C2 or C4. She can also resupply in port (INREP), or underway (UNREP), as can all other surface ships. In addition to surface combatants, this catalog includes attack submarines, and noncombatant naval ships that might optionally require defensive combatant escorts (e.g., amphibious assault ships, mine sweepers and supply ships). If we include logistics support features in our scenario, other tables not shown here give for each ship her capacity, initial inventory and usage rate for fuel and other commodities. These ships all start service on day *d*1 because this is a preplanned operation; other plans may give us operational control of ships on a staggered schedule. Although LCS can sprint at high speed, they are restricted here to 15 knots underway to conserve fuel, most other ships will make 24 knots.

Table A.5. Combined mission capability (CMC) sets.

CMC	IAMD	ASW	SUW	STRIKE	NSFS	MIO	MIW	LOG	INTEL	LIFT	UNREP	ESCORT
C1	1		1	1	0.5				1			
C2	1	1	1	1	0.5				1			
C3		1	0.5	0.5	0.5	1			1			
C4	1	1	1						1		1	1
C5	1	1	1						1			1
C6	1	0.75	1	1	0.5				1			
C7	1	1	1	1					1		1	
C8	1				1							
C9	1	0.5	0.5	1	1							
C10		1	1			1					1	1
C11		1	0.5			1					1	1
C12			0.5				1		1			
C13			0.5			1	1					
C14											1	
C15								0.5		1	1	
C16								0.5		1	1	
C17								0.5		1	1	
C18						1					1	
C19		1	1	1								
C20		1										

Notes: A guided missile cruiser such as CG61 Monterey might operate in CMC C2 when it can simultaneously conduct missions IAMD, ASW, SUW, STRIKE, NSFS, and INTEL; however, in this mode her NSFS mission effectiveness is degraded to 50% accomplishment, meaning NSFS mission completion would require assistance from some other ship(s). There may be more than one CMC containing the same missions that might apply to the same ship class: this would signify that some ships are in a better readiness state than others. There are also CMCs not shown for states engaging in no mission, but consuming commodities, such as one for any day of TRANSIT, and one for a ship rendered out of commission (OOC).

Table A.6. Employment schedule for DDG112 Michael Murphy, Case C.

DDG112 Michael Murphy; days between replenishments: min 3, max 7; 24 knots; collocated ships, CMCs, and missions			
Day	Region	CMC	Mission(s)
*d*1	Guam		
*d*2	Guam	C_IN	INREP
*d*3	*r*4	C7	IAMD, ASW
*d*4	*r*4	C7	IAMD, ASW
*d*5	Guam	C_IN	INREP
*d*6	*r*5	C7	IAMD, ASW
*d*7	*r*5	C7	IAMD, ASW
*d*8	*r*4	C7	IAMD, ASW
*d*9	Guam	C_IN	INREP
*d*10	*r*_TR		
*d*11	*r*_TR		
*d*12	*r*8	C7	IAMD, ASW
*d*13	*r*8	C7	IAMD, ASW
*d*14	*r*8	C7	IAMD, ASW

Notes: On day *d*2 this guided missile destroyer is in Guam preparing to deploy as an escort of a task group headed west. She provides IAMD and ASW protection in the area around Guam, subsequently moving further west, then returning. This unintuitive action turns out to be an optimal combination of mission engagement and refueling.